A Simplified Plasma Current Profile Model for Tokamak Control

E. Witrant, S. Brémond, F. Delebecque and J.-P. Quadrat

***Abstract*— The purpose of this paper is to present a simplified model of the current and temperature dynamics of tokamak plasma. It is focused on the diffusion behaviors and relates the profiles of physical variables to engineering control inputs. The Scilab/Scicos environment is used for the numerical implementation of this model. This work is a first step towards the control of the current profile.**

I. INTRODUCTION

A tokamak is a physical device in which a plasma is confined using magnetic coils set in the poloidal and toroidal planes (see Figure 1). The plasma behaves as a conductor that is heated by the current induced by the variation of the magnetic flux in the ohmic coils. The tokamak can then be considered, in a first approximation, as a large transformer where the current of the secondary coils is used to heat the primary coil. As the plasma resistivity is decreasing with temperature, it is also necessary to add other heating sources that enhance the plasma confinment (confinment of the energy at the center of the plasma) and increase the overall temperature. This is done thanks to radio-frequency antennas (such as the lower hybrid one considered in this work) that allow to reach a very high central temperature, which is necessary to obtain fusion reactions.

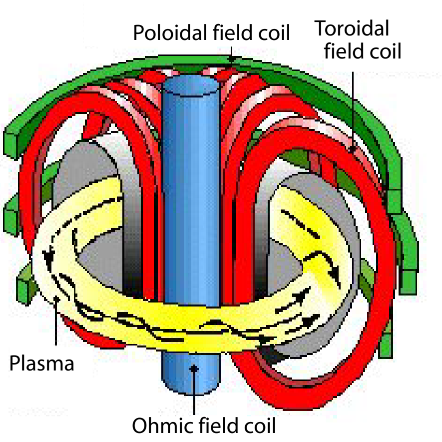

Fig. 1. Tokamak.

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The control of tokamak plasma has a long history (see [3]-[4]). In particular, four classes of control problems have been investigated:

- vertical stabilization of the plasma center,
- control of the magnetic surfaces shape,
- control of mangnetohydrodynamic (MHD) instabilities,
- control of the current, temperature and density profiles.

We are concerned with the last problem, which has been studied more recently in [6], [5], [7]. The goal is to provide for the operating conditions (in terms of profiles shapes) that are necessary to achieve advanced confinment schemes and increase the fusion power production efficiency. For example, the so called H-mode is characterized by a transport barrier located at the plasma edge, which improves the confinment, and will be the operating mode of the future ITER tokamak.

The previous profile control approaches cited above are mainly based on black box linear models and plasma physics are only used to select the set of relevant variables and the way they are coupled. These approaches imply the identification of a MIMO system approximating a distributed system and are highly dependant on the operating conditions, which makes them costly in terms of experimentations. The aim of this paper is then to provide for a control-oriented model established with a nonlinear system of PDE based on:

- the evolution of the resistive equation averaged on the magnetic surface as explained in [1],
- the experimental identification of some diffusion coefficients.

The control problem is formulated and the model giving the current density is numerically solved in the Scilab-Scicos environment. Based on some experimental data, the simulation results provided by the proposed model are in good agreement with those obtained by the Cronos software. Cronos is one of the references to study the transport equations in tokamak plasmas and includes complex physical knowledge, but it can not be used in real-time or for control purposes.

In the second section, we recall some useful plasma physics principles and the averaging method used to obtain the resistive equation. The resistive model is completed in the third section by specifying the resistivity and the non-inductive current sources, as given in [9]. Then the complete model is solved in Scilab and compared with Cronos corresponding results. In the last section, the current profile control problem is set and briefly discussed.

II. Tokamak plasma physics

We recall here some basic physics notions used to model the plasma in modern Tokamaks.

A. Plasma magnetohydrodynamics

The dynamics of a plasma is governed by (see [1], [8]) the MHD equations:

$$\begin{cases} \nabla \times E = -\partial_t B, \quad \text{Faraday's law,} \\ E + \zeta j_n + u \times B = \zeta j, \quad \text{Ohm's law,} \\ \nabla . B = 0, \quad \text{conservation of B,} \\ \nabla \times B = \mu_0 j, \quad \text{Ampère's law,} \\ \partial_t n + \nabla.(nu) = n_s, \quad \text{particles conservation,} \\ mn\,\dot{u} + \nabla p = j \times B, \quad \text{momentum conservation,} \\ \frac{3}{2}\,\dot{p} + \frac{5}{2} p \nabla . u + \nabla . Q = p_s, \quad \text{energy conservation,} \\ p = knT, \quad \text{perfect gases law,} \end{cases}$$

where $\dot{v} \triangleq \partial_t v + v.\nabla v$, E is the electric field, B is the magnetic field, u is the mean particles velocity, j is the current density, j_n is the non inductive current density, n is the particles density, p is the plasma pressure, T is the temperature, Q is the heat flux due to particle collisions, m is the particle mass, μ is the magnetic permeability, ζ is the resistivity tensor, k is the Boltzmann constant, n_s is the particle source and p_s is the energy source.

B. Time Constants

In order to model the plasma behavior, it is important to understand the different time constants associated with the physical phenomena. We can discern four time constants:

- The Alfvén time $\tau_A = a(\mu_0 mn)^{1/2}/B_0$, where a is the minor radius of the tore and the subscript 0 denotes the physical value at the plasma center, is of the order of $10^{-6}s$ for ions and $10^{-9}s$ for electrons;
- The density diffusion time $\tau_n = a^2/D$, where D is the particle diffusion coefficient, is of the order of 10^{-3}s;
- The heat diffusion time $\tau = na^2/K$, where K is the thermal conductivity of particles, is of the order of 10^{-3}s;
- The resistive time constant $\tau_r = \mu_0 a^2/\zeta$ is of the order of $1s$.

The Alfvén time scale is used to describe the MHD instabilities phenomena, which are not considered here. Our model is focused on the dynamics of the resistive behavior of the plasma. Due to the differences in the time scales, only the global, steady-state time variations of temperature and density are then included in the model.

C. Magnetic Surfaces

In this paper we are interested in the dynamics of the current density profile, i.e., phenomenas having a time constant of 1s. At this time scale, we can consider that the the momentum equation is at the equilibrium i.e. :

$$\nabla p = j \times B\ .$$

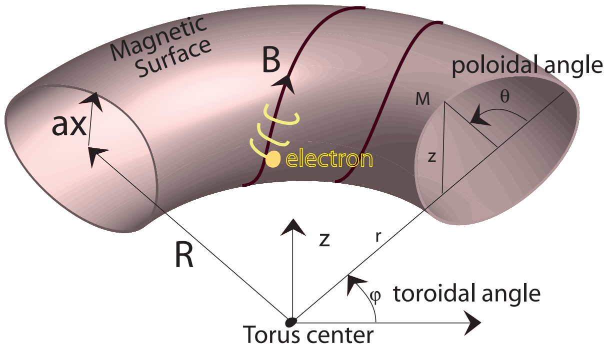

Fig. 2. Magnetic Surface.

This equation yields $B.\nabla p = 0$ and $j.\nabla p = 0$, and therefore the magnetic field lines and the current lines lie in the so called *magnetic surface* which are surfaces of constant pressure. The magnetic surfaces form a set of nested toroids indexed by x, presented in Figure 2.

D. Poloidal Magnetic Flux and Current Flux

From the conservation of B, it follows that there exists A such that $B = \nabla \times A$ with $A = (A_r, A_\varphi, A_z)$. From the tore symmetry, A is independent of the toroidal angle φ. Therefore $B_r = -(1/r)\partial_z(rA_\varphi)$ and $B_z = (1/r)\partial_r(rA_\varphi)$. In the following rA_φ will be denoted Ψ.

The tore symmetry implies that $\partial_\varphi p = 0$. The magnetic field B being orthogonal to ∇p we have $-\partial_r p \partial_z \Psi + \partial_z p \partial_r \Psi = 0$ which means that ∇p is proportional to $\nabla \Psi$, thus Ψ is constant on a magnetic surface.

Denoting by $\mathcal{D}(r,z)$ the horizontal disk centered on the z-axis with its boundary passing trough the point M (with coordinates (r,z)), the Stoke's formula applied to $\mathcal{D}$ and the field B :

$$2\pi\Psi = 2\pi r A_\varphi = \int_{\partial\mathcal{D}} A = \int_{\mathcal{D}} \partial A = \int_{\mathcal{D}} B\ ,$$

gives the interpretation of Ψ as the poloidal magnetic flux. Similarly, applying Stoke's to $\mathcal{D}$ and the field j, using Ampère's law and denoting rB_φ by f we have:

$$2\pi f = 2\pi r B_\varphi = \int_{\partial\mathcal{D}} B = \int_{\mathcal{D}} \partial B = \mu_0 \int_{\mathcal{D}} j\ .$$

We can show using the orthogonality of j with ∇p that f is constant on the magnetic surfaces, as it has been done for Ψ.

From Ampère's law and the definition of Ψ we have:

$$j_\varphi = \frac{\partial_z B_r - \partial_r B_z}{\mu_0} = L\Psi \triangleq -\frac{1}{\mu_0}\Big(\partial_z\Big(\frac{\partial_z \Psi}{r}\Big) + \partial_r\Big(\frac{\partial_r \Psi}{r}\Big)\Big)\ .$$

To summarize,

$$B = \Big(-\frac{\partial_z \Psi}{r}, \frac{f}{r}, \frac{\partial_r \Psi}{r}\Big)\ , \quad (1)$$

$$j = \Big(-\frac{\partial_z f}{\mu_0 r}, L\Psi, \frac{\partial_r f}{\mu_0 r}\Big)\ . \quad (2)$$

E. Grad-Shafranov Equation

Using (1),(2) and the colinearity of $\nabla\Psi$ and ∇f,

$$\nabla p = j \times B = \frac{L\Psi}{r}\nabla\Psi - \frac{f}{\mu_0 r^2}\nabla f,$$

which gives the Grad-Shafranov equation :

$$L\Psi = r\partial_\Psi p + \frac{1}{2\mu_0 r}\partial_\Psi(f^2)\ .$$

F. Mean magnetic radius

We define the mean geometric radius of magnetic surface, denoted by ρ, as

$$\rho \triangleq \sqrt{\frac{\Phi}{\pi B_0}}\ , \tag{3}$$

where B_0 is the magnetic field at the center (which is purely toroidal and assumed to be constant) and

$$\Phi \triangleq \int_{\mathcal{S}} Bd\mathcal{S} = \frac{1}{2\pi}\int_{\mathcal{V}}\frac{B_\varphi}{r}d\mathcal{V} = \frac{1}{2\pi}\int_{\mathcal{V}}\frac{f}{r^2}d\mathcal{V}\ , \tag{4}$$

where $\mathcal{S}$ denotes a poloidal section of a magnetic surface and $\mathcal{V}$ the volume enclosed by this magnetic surface. In the sequel, we will assume that the magnetic surfaces are time constant, that $\mathcal{S}$ is a disk, and that $\varepsilon \triangleq \rho/R$ (where R is the *major radius*) is small.

G. Security Factor

The *security factor* is defined by

$$q \triangleq -\frac{1}{2\pi}\frac{\partial\Phi}{\partial\Psi}\ .$$

It is is equal to B_φ/B_θ where B_θ is the poloidal magnetic field and $B_\theta = \sqrt{B_r^2 + B_z^2}$. Higher values of q lead to greater plasma stability, thus it is an important output plasma variable.

H. Resistive Diffusion Equation

Applying the Stoke's formula to the Faraday's equation gives :

$$2\pi r E_\varphi = \int_{\partial\mathcal{D}} E = \int_{\mathcal{D}}\partial E = -\int_{\mathcal{D}}\partial_t B$$
$$= -\partial_t\int_{\mathcal{D}} B = -2\pi\partial_t\Psi\ .$$

Using $B_\varphi = f/r$, we have $\partial_t\Psi = -r^2 E_\varphi B_\varphi/f$.

Note that since Ψ and f are constant on each magnetic surface there exists $\bar\Psi$ and $\bar f$ such that $\Psi(r,z) = \bar\Psi(\rho(r,z))$ and $f(r,z) = \bar f(\rho(r,z))$.

Assuming that $\partial_t\rho = 0$ it can be shown after some calculation (see [1], [2]) that

$$\partial_t\bar\Psi = -\frac{\langle E.B\rangle}{f\ \langle 1/r^2\rangle}$$

Now, using the Ohm's law we have $\langle E.B\rangle = \eta\langle(j - j_n).B\rangle$ and therefore

$$\partial_t\bar\Psi = -\frac{\eta\langle(j - j_n).B\rangle}{f\ \langle 1/r^2\rangle}\ ,$$

where : – $\langle A\rangle \triangleq \partial_{\mathcal{V}}\int_{\mathcal{V}} Ad\mathcal{V}$ with $\mathcal{V}$ the volume inside the magnetic surface, – η denotes the component of ζ parallel to the magnetic surface.

Denoting $v' = \partial_\rho\mathcal{V}$, since

$$\langle\nabla.A\rangle = \partial_{\mathcal{V}}\langle A.\nabla\mathcal{V}\rangle = \frac{1}{v'}\partial_\rho(v'\langle A.\nabla\rho\rangle)\ ,$$

we have :

$$\langle(j - j_n).B\rangle = \langle\frac{1}{\mu_0 r^2}(\partial_r\Psi\partial_r f + \partial_z\Psi\partial_z f) + \frac{f}{r}L\Psi\rangle$$
$$= \frac{\langle|\nabla\rho|^2/r^2\rangle}{\mu_0}\partial_\rho\bar\Psi\partial_\rho\bar f - \frac{\bar f}{\mu_0 v'}\partial_\rho\Big(v'\langle|\nabla\rho|^2/r^2\rangle\partial_\rho\bar\Psi\Big)\ ,$$
$$= -\frac{\bar f^2}{\mu_0 v'}\partial_\rho\Big(v'\langle|\nabla\rho|^2/(\bar f r^2)\rangle\partial_\rho\bar\Psi\Big)\ .$$

Therefore we obtain :

$$\partial_t\bar\Psi = \frac{\eta\bar f}{\mu_0 c_3}\partial_\rho\Big(\frac{c_2}{\bar f}\partial_\rho\bar\Psi\Big) + \frac{\eta\langle j_n.B\rangle}{\bar f\langle 1/r^2\rangle}\ , \tag{5}$$

with

$$c_2(\rho) = v'\langle|\nabla\rho|^2/r^2\rangle,\quad c_3(\rho) = v'\langle 1/r^2\rangle\ .$$

Using (4) and (3) we have :

$$\partial_\rho\Phi = \frac{\bar f v'}{2\pi}\langle 1/r^2\rangle = 2\pi\rho B_0\ ,$$

and therefore

$$\bar f = \frac{4\pi^2\rho B_0}{c_3}.$$

Substituting f by its value in (5) we obtain the *resistive equation* :

$$\partial_t\bar\Psi = \frac{\eta\bar f}{\mu_0 c_3^2}\partial_\rho\Big(\frac{c_2 c_3}{\rho}\partial_\rho\bar\Psi\Big) + \frac{\eta v'\langle j_n.B\rangle}{4\pi^2\rho B_0}\ . \tag{6}$$

By symmetry, the boundary condition at $\rho = 0$ is

$$\partial_\rho\bar\Psi(0) = 0\,. \tag{7}$$

The boundary condition at $\rho = \rho_{\max}$ is obtained by computing I, the total toroidal plasma current :

$$I = \int_{\mathcal{S}} j_\varphi d\mathcal{S} = \frac{1}{2\pi}\int_{\mathcal{V}}\langle j_\varphi/r\rangle d\mathcal{V} = \frac{1}{2\pi\mu_0}\int_{\mathcal{V}}\langle L\Psi/r\rangle d\mathcal{V}$$
$$= -\frac{1}{2\pi\mu_0}\int_\rho\partial_\rho\Big(v'\langle|\nabla\rho|^2/r^2\rangle\partial_\rho\bar\Psi\Big)d\rho = -\frac{c_2\partial_\rho\bar\Psi(\rho_{\max})}{2\pi\mu_0}\ .$$

Therefore :

$$\partial_\rho\bar\Psi(\rho_{\max}) = \frac{-2\pi\mu_0 I}{c_2}\ . \tag{8}$$

III. RESOLUTION OF DIFFUSION RESISTIVE MODEL

Here we specify the resistive model (6), that is :

- We give empirical formula for the resistivity η, bootstrap current and hybrid antenna current deposit (which are the only two sources of non inductive current considered),
- We assume that ε is small (cylindrical assumption) and that the mean small radius is time constant $\rho = ax$.

We solve the corresponding resistive equation using the ODE solver of Scilab-Scicos and compare the results obtained with those computed by Cronos. This model has been introduced and discussed in more detailed in [9].

The empirical scale laws given here are based on the Tore Supra experiments and have not been validated on other tokamaks.

	Primitive Constants
R	major radius of the plasma (m)
a	minor radius of the plasma
e	electric electron charge
Z	effective ion electron charge ratio
m_e	electron mass
m_i	average ion mass (kg)
μ_0	permeability of free space (H/m)
ε_0	permittivity of free space (F/m)
	Derived Constants
ε	a/R inverse aspect ratio
v	$2\pi^2 a^2 R$ tore volume
c_j	$2\pi^2/\mu_0 v$
c_I	$R\mu_0/2\pi$
c_q	$a^2 B_0$
c_v	$\pi 10^6/v$
c_ν	$R\sqrt{m_e}/\varepsilon^{1.5}$
c_T	$6\sqrt{2}\pi^{3/2}\varepsilon_0^2/(e^4\sqrt{m_e})$
c_D	$3m_e/(m_i\tau_e)$
	State Related Variables
T_e	electron temperature profile(J)
T_i	ion temperature profile(J)
α	$(1-T_i/T_e)$ ion electron temperature ratio profile
n_e	electron density profile
n_i	ion density profile
$\bar{n}$	space average of electron density
τ_e	electron collision time
τ_t	thermal energy confinement time
ν_e	electron collisionality parameter
η	plasma resistivity profile
B_φ	toroidal magnetic field profile
$\bar{\Psi}$	magnetic flux profile of the poloidal field
j_b	bootstrap current density profile
j_h	hybrid current density profile
j_φ	toroidal current density profile
χ_e	electronic temperature diffusion
p	total power
p_Ω	ohmic power
	Input Related Variables
I	total plasma current (A)
j_h	hybrid current profile
θ_h	maximal hybrid current deposit
p_h	hybrid antenna power
n_h	parallel refraction index
m_h	maximum hybrid deposit location
v_h	variance of hybrid deposit location
ρ_h	heat/power proportion of hybrid antenna deposit

	Output Related Variables
q	safety factor profile
	Composition Variables
c_L	$3.3735\,10^{-33}Z(0.73+0.27Z)/(0.53+0.47Z)$
c_r	$0.56(3-Z)/(Z(3+Z))$
c_ξ	$0.58+0.2Z$
c_h	$1.18Z^{-0.24}$
d_0	$1.414Z+Z^2$
d_1	$0.754+2.657Z+2Z^2$
d_2	$0.348+1.243Z+Z^2$
a_{10}	$0.754+2.21Z+Z^2$
a_{11}	d_2
a_{20}	$0.884+2.074Z$
	Shape Variables
ζ	εx
f	$1-(1-\zeta)^2/\big(1+1.46\sqrt{\zeta}\big)\sqrt{(1-\zeta^2)}\big)$
r	$f/(1-f)$ ratio of trapped to circulating particles
d	$d_0+d_1 r+d_2 r^2$
a_1	$r(a_{10}+ra_{11})/d$
a_2	$a_{20}r/d$

When only hybrid effect antennas are used, the dynamic equation of the magnetic flux is :

$$\begin{cases}\partial_t\bar{\Psi} = R\eta(\partial_x\bar{\Psi},t)\big(j_b(\partial_x\bar{\Psi},t)+j_h(t)+c_j\frac{1}{x}\partial_x(x\partial_x\bar{\Psi})\big),\\ \partial_x\bar{\Psi}(t,0)=0,\quad \partial_x\bar{\Psi}(t,1)=c_I I(t),\end{cases} \tag{9}$$

where :

- $-c_j\frac{1}{x}\partial_x(x\partial_x\bar{\Psi})$, denoted $j_\varphi(\bar{\Psi},t)$, is the toroidal current plasma profile,
- j_h is the current deposit coming from the lower hybrid effect antenna given below,
- j_b is the bootstrap current described later,
- c_j is a constant,
- I is a total plasma current.

The security factor can be rewritten as :

$$q(t,x) = \frac{-c_q x}{\partial_x\bar{\Psi}(t,x)}\ . \tag{10}$$

Typically, we control the density profile using the hybrid current deposit $j_h(p_h,n_h)$ (through the control variables p_h and n_h) and the total current I. We would like obtain and stabilize a specified security factor profile q at appropriate fusion conditions.

A. Resistivity

The resistivity η is a function of : – q (which depends of $\partial_x\Psi$), – n_e, n_i, T_e and T_i which are considered here as given time functions (in fact given by the Cronos software) :

$$\Lambda = 31.3181 + \log\big(T_e/(e\sqrt{n_e})\big), \tag{11}$$

$$\tau_e = c_T T_e^{3/2}/(\Lambda n_e), \tag{12}$$

$$\nu = c_\nu x^{3/2} q/(\tau_e\sqrt{T_e}), \tag{13}$$

$$w = f/(1+c_\xi\nu), \tag{14}$$

$$\eta = c_L\Lambda/\big(T_e^{3/2}(1-w)(1-c_r w)\big). \tag{15}$$

B. Bootstrap Current

The bootstrap current comes from a complex mechanism where some particles do not follow the magnetic field but are trapped in a plasma zone. The contribution of the electron (only considered here) to the induce current is given by :

$$j_b = R\frac{(a_1 - a_2)n_e\partial_x T_e + a_1 T_e \partial_x n_e}{\partial_x \Psi} .$$

C. Lower Hybrid Current Deposit

The hybrid antenna current deposit has a shape which can be approximated by a truncated (on positive real numbers) gaussian density with mean m_h and variance v_h defined by :

$$\begin{aligned} M_h &= 0.5354^{-0.2446} I^{0.5723} \bar{n}^{-0.0789} p_h^{0.1337} n_h^{0.3879} , \\ m_h &= 0.1985 B^{-0.3905} I^{0.7061} \bar{n}^{-0.0178} p_h^{0.126} n_h^{1.1974} , \\ v_h &= (m_h - M_h)^2/(2\log(2)) . \end{aligned}$$

The total current deposit I_h is given by :

$$\begin{aligned} \eta_h &= c_h(2.03 - 0.63 n_h)^{0.55} I^{0.43} , \\ I_h &= \eta_h p_h / \bar{n} , \end{aligned}$$

and other empirical formulas are also available.

Normalizing the shape deposit by its average c_g we obtain the hybrid deposit j_h :

$$\begin{aligned} c_g &= m_h\sqrt{\frac{v_h\pi}{2}}\left\{\text{erf}\left(\frac{1-m_h}{\sqrt{2v_h}}\right) - \text{erf}\left(\frac{-m_h}{\sqrt{2v_h}}\right)\right\} \\ &+ v_h\left\{\exp\left(\frac{-m_h^2}{2v_h}\right) - \exp\left(\frac{-(1-m_h)^2}{2v_h}\right)\right\} , \\ j_h &= I_h\frac{c_v}{c_g}\exp\left(\frac{-(x-m_h)^2}{2v_h}\right) . \end{aligned}$$

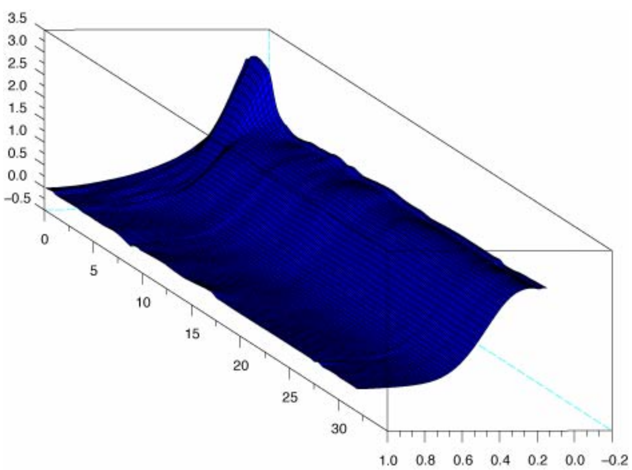

Fig. 3. Current density.

D. Scilab/Scicos Implementation

This resistive model has been solved numerically using the scientific free software Scilab. The equation is solved using the default ode solver of Scilab. The state derivatives are approximated by appropriate differentiation matices. The simulation can be done by a script function or implemented using the Scicos block-diagram editor (see Figure4). The ode solver uses multistep BDF formulas and the numerical results are obtained within a few seconds.

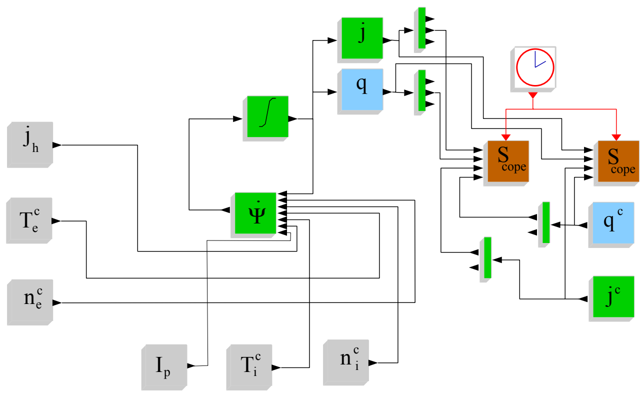

Fig. 4. Scicos Diagram for Magnetic Flux Integration.

The numerical results obtained are compared to those obtained using Cronos which is a set of Matlab programs dedicated to the simulation of the plasma transport equation and the description of the actuator interaction with the plasma. The results given here are obtained using the data of Cronos for states which are still not modeled in this simplified model, in particular the evolution of plasma temperature and electron density. There is an important drift between the $\bar{\Psi}$ obtained the one of Cronos, but this not affect the quality of the current variable (as it can be seen in Figure 5) and the security factor one (Figure 6). The results are not so good for the bootstrap current (Figure 5).

IV. PRELIMINARY REMARKS ON PROFILE CONTROL AND IDENTIFICATION

A. Identification

Some functions in the previous model are of multivariable monomial type with unknown exponents estimated offline. We can try to estimate on line some of these important exponents. Typically we have to estimate α in a diffusion equation such as :

$$\begin{cases} \partial_t T = \partial_x(T^\alpha \partial_x T) + u, \\ \partial_x T(t,0) = 0, \\ T(t,1) = T_1. \end{cases}$$

For this simplified model, it is easy to build a convergent observer : indeed integrating from 0 to x the equation we

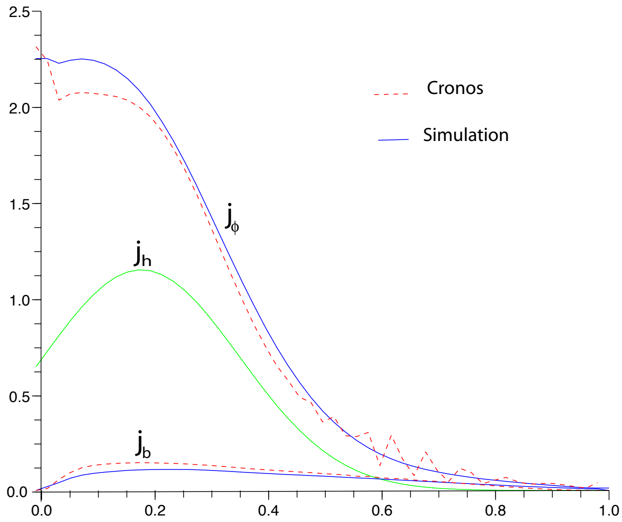

Fig. 5. Current density profiles.

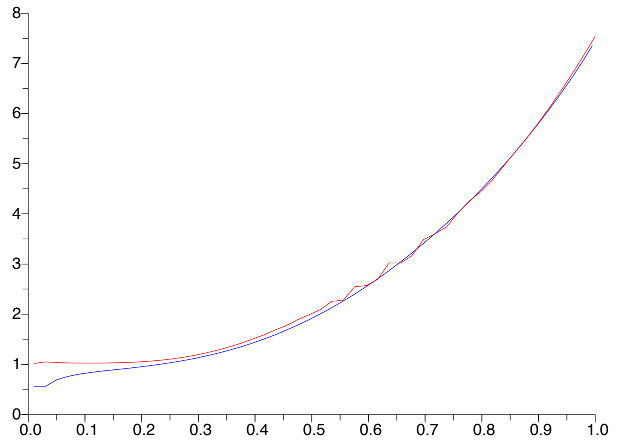

Fig. 6. Security factor profile (q) : – simplified model (dash), – Cronos (solid).

obtain :

$$\partial_t \int_0^x T = (T^\alpha \partial_x T)(x) + \int_0^x u$$

and this leads to the estimate :

$$\dot{\alpha} = k \log T\Big(\log \frac{\partial_t \int_0^x (T-u)}{\partial_x T(x)} - \alpha \log T \Big).$$

B. Formulation of the current profile control problem

In the problem considered here, the control variables are I, p_h and n_h. We will consider in the following that, by adding additional antennas, an arbitrary deposit j_h can be realized. The variables B_0 and $n_e(t,x)$ are supposed to be given (in fact n_e satisfies a transport equation which has a diffusion coefficient which is difficult to model).

The controls I and j_h have an influence on the temperature which can be considered as an external input. The

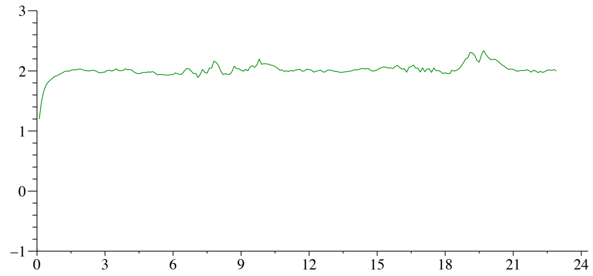

Fig. 7. Estimator Convergence ($\alpha = 2$, uniform noise belonging to $[-0.4, 0.4]$ on the derivative).

dynamic of the electronic temperature $T_e(t,x)$ being much faster than the magnetic flux dynamics, we can assume that the temperature is at its equilibrium. At each time, it satisfies the following static diffusion equation (which can be derived as it has be done for the equation of $\bar{\Psi}$ see [1]) :

$$\begin{cases} 0 = \dfrac{1}{a^2 x}\partial_x(x n_e \chi_e \partial_x T_e) + \eta j_\varphi (j_\varphi - j_h - j_b) \\ \qquad\qquad\qquad -c_D n_e T_e \alpha + \rho_h j_h \ , \\ \partial_x T_e(t,0) = 0 \ , \\ T_e(t,1) = \theta \text{ given} \ , \end{cases} \tag{16}$$

with :

$$\chi_e = 4.68 \frac{a q^2}{B_0} \frac{|\nabla(n_e T_e)|}{n_e} \ ,$$

$\rho_h j_h$ the hybrid heat deposit proportional to j_h and T_i the ion temperature given by the empirical formula (see [10]) :

$$\alpha = 0.31 \left(\frac{I}{B_0}\right)^{-0.38} \bar{n}^{-0.90} \left(1 + \frac{p_h}{p}\right)^{1.36}$$

where $p \triangleq p_\Omega + p_h$ is the total power that is the sum of the ohmic power $p_\Omega = \int_0^1 \eta j_\varphi^2 dx$ and the hybrid antenna power p_h.

We can take $\bar{\Psi}$ as the state of the system. Indeed, given $\bar{\Psi}$ and the controls, we can compute j_φ from (9), q from (10) and then compute T_e by solving the static equation (16). Therefore T_e is a function of $\bar{\Psi}$ and the controls. Then substituting it in η (15), we obtain, in principle, explicit dynamics for $\bar{\Psi}$.

The purpose of the control is to obtain a given profile for the security factor q and maintain it during a given period of time. Clearly, there exists a family of $\bar{\Psi}$ giving a specific q profile. Denoting by $\bar{\Psi}_q$ such a flux profile and using (9) at the equilibrium $\partial_t \bar{\Psi} = 0$, we see that to obtain the desired flux profile we have to solve a set of equations with the following structure :

$$F(\bar{\Psi}_q, T_e, I) = j_h \ ,$$
$$G(\bar{\Psi}_q, T_e, I) = j_h \ .$$

Here F is derived from (9) and G from (16). Therefore, if there exists T_e^q satisfying $F(\bar{\Psi}_q, T_e^q) = G(\bar{\Psi}_q, T_e^q)$ the above compatibility conditions are satisfied and the corresponding control j_h gives the desired equilibrium around which the system should be stabilized.

The variable $\bar{\Psi}$ is not observed but we are able to observe T_e. From T_e and the control, using (16) (where we see $\bar{\Psi}$ as unknown) and the boundary conditions of (9), we can compute $\bar{\Psi}$.

Summarizing, supposing that we have enough antenna to be able to approximate any deposit profile, we have to control a nonlinear system where we can consider that : – we observe the state, – the system is controllable

V. CONCLUSION

The nonlinear model given here aims at providing reliable simulation results. As such it can be used to validate control laws. Based on the profile obtained it seems that we can obtain simpler nonlinear model to design the control law.

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